Determine if $y = Ax + B\ln x + \frac{1}{x}$ is a family of solutions of the DE $(x^2 - x^2\ln x)y'' + xy' - y = \frac{2\ln x}{x}$. **SCORE: _____ / 6 PTS**

State your conclusion clearly.

$$(x^2 - x^2\ln x)\left(-\frac{B}{x^2} + \frac{2}{x^3}\right) \quad 1$$
$$+ x\left(A + \frac{B}{x} - \frac{1}{x^2}\right) \quad 1$$
$$-\left(Ax + B\ln x + \frac{1}{x}\right) \qquad 1$$

$$= -B + B\ln x + \frac{2}{x} - \frac{2\ln x}{x}$$
$$+ Ax + B \qquad - \frac{1}{x}$$
$$- Ax \qquad - B\ln x \quad - \frac{1}{x}$$

$$= -\frac{2\ln x}{x} \neq \frac{2\ln x}{x}$$

NOT A FAMILY OF SOLUTIONS

2 1

What does the Existence and Uniqueness Theorem tell you about possible solutions to the IVP **SCORE: _____ / 4 PTS**

$x+(y')^3=1+\sin y, \quad y(1)=\pi$? **Justify your answer properly, but briefly.**

$\underbrace{y'=\underbrace{(1-x+\sin y)^{\frac{1}{3}}}_{f}}$ CONT AROUND $(1,\pi)$ [ACTUALLY EVERYWHERE]

(1)

(1) $f_y=\frac{1}{3}(1-x+\sin y)^{-\frac{2}{3}}\cdot\cos y$, NOT CONT. @ $(1,\pi)$ (1)

SINCE $1-1+\sin\pi=0$

$0^{-\frac{2}{3}}$ UNDEFINED

(1) NO CONCLUSION

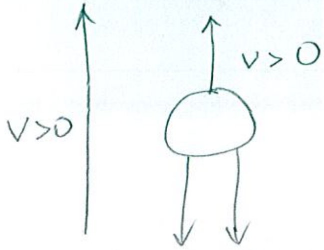
An object of mass m kilograms is shot upward at high speed. Its velocity is affected by the forces of gravity and air resistance. Because of the high speeds involved, the force of air resistance is proportional to the square of the velocity. (Assume $v > 0$ corresponds to upward motion.)

SCORE: _____ / 4 PTS

Write a differential equation for the velocity of the object as it rises. All symbolic constants in your differential equation must represent positive numbers. You may use the symbolic constant g to represent $9.8 \frac{m}{s^2}$. **<u>Do NOT use the absolute value function in your answer.</u>**

$$m\frac{dv}{dt} = F_{GRAVITY} + F_{AIR}$$

$$\underbrace{m\frac{dv}{dt}}_{1} = \underbrace{-mg}_{1} \underbrace{-kv^2}_{2}$$



$F_{AIR} < 0 \quad F_{GRAVITY} < 0$

$kv^2 > 0 \rightarrow k < 0$

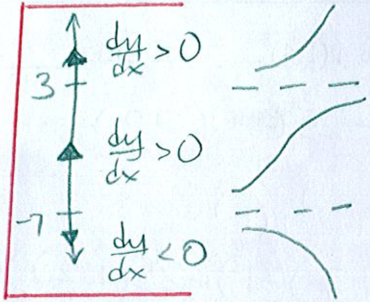
Consider the DE $\frac{dy}{dx} = (7+y)^3(3-y)^2$. **SCORE: _____ / 6 PTS**

[a] Find all equilibrium solutions of the DE and classify each as stable, unstable or semi-stable.
<u>You must draw a phase portrait to get full credit.</u>

$\frac{dy}{dx} = 0$ @ $y = -7, 3$

MUST HAVE "$y=$"

2



$y=3$, SEMI-STABLE

$\frac{1}{2}$ 1

$y=-7$, UNSTABLE

$\frac{1}{2}$ 1

[b] If $y = m(x)$ is a solution of the DE such that $m(-8) = -5$, what is $\lim_{x\to\infty} m(x)$?

3

1

Consider the IVP $y' = 10xy + 25x$, $y(1) = -2$. Use Euler's method with $h = 0.2$ to estimate $y(1.4)$. **SCORE: _____ / 4 PTS**

$$y(1.2) \approx y(1) + y'(1)(1.2-1) = -2 + (10(1)(-2) + 25(1))(0.2)$$

$$= \underbrace{-2 + 5(0.2)}_{1} = \underbrace{-1}_{1}$$

$$y(1.4) \approx y(1.2) + y'(1.2)(1.4-1.2) \approx -1 + (10(1.2)(-1) + 25(1.2))(0.2)$$

$$\approx \underbrace{-1 + 18(0.2)}_{1} = \underbrace{2.6}_{1}$$